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SAMPLING STUDIES of the CLOVER ROOT BORER

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INTRODUCTION

The evaluation of populations is an important part of most investigations on the ecology or control of an insect. Evaluation of insect populations is usually complicated by a tendency of the species to be non normally distributed. The clover root borer, _Hylastinus obscurus_ (Marsh.) shows a decided tendency in this regard.

As pointed out by McGuire _et al._ (1957) a study of distributions is usually made for one of two reasons: a) to find a transformation in order to use normal theory in performing statistical tests; or b) to relate the observed counts to some theory of population growth or spread. The primary purpose of this study was to find a transformation suitable for root borer counts so that normal theory could be used in statistical tests. The general approach to the problem was to fit several sample distributions to theoretical mathematical models which are currently available. After a model distribution was decided upon, transformations that have been suggested for such a model were applied to sample data. The criterion for judging the adequacy of the transformation was the freedom from correlation of the mean and the variance. Other characteristics of the transformed data were investigated.

1/ Research Assistant and Associate Entomologist respectively. The senior author is presently Assistant Entomologist, Nebraska Agricultural Experiment Station. The junior author is now Station Statistician, Ohio Agricultural Experiment Station.

FITTING THE DISTRIBUTIONS

The sampling unit employed was the clover root. The number of borers - egg, larvae, pupae, new and old adults - were counted in each root. In the studies reported here only the larvae, pupae, and new adult forms were included and they were lumped together in a single count for each root. Three years sampling data were available from various experiments in Ohio including extensive observations from a statewide survey made in 1954. Several sampling studies gave additional information on the distribution of 5 and 10 root samples, as well as the distribution of borers in individual roots. These data were supplemented with borer counts supplied by R. T. Everly of Indiana, C. S. Koehler of New York, E. A. Dickason of Oregon, and A. M. Woodside of Virginia. The assistance of these workers is gratefully acknowledged.

The distributions usually considered for this type of data are some sort of compound Poisson. In the case of the root borer data the negative binomial, several of Neyman's distributions, and a "double Poisson" were tested against the observed distributions. A typical distribution and the calculated expected distributions are illustrated in table 1. In all, 26 separate sampling distributions were investigated and fitted to one or another of Neyman's distributions and to the negative binomial. A chi square goodness of fit test was applied to the observed and expected data. The distribution of the chi square probabilities of 23 sets of sample data is listed in table 2.

Obviously the Neyman's distributions fit better than the negative binomial, although they still had too many cases with low probability of occurrence. In general only the Neyman's $n = 0$ and $n \rightarrow \infty$ were used. The fit might have been improved by a more judicious choice of n or by estimating parameters by the method of frequency of zeros and ones in the case of samples with a few unusually high counts. In the latter case, the high counts lead to an especially high estimate of variance.

The negative binomial and Neyman's distributions were fitted by the method of moments (Bliss, 1953 and Beall and Rescia, 1953.) The double Poisson was fitted by a method described by Thomas (1949). Since this study was completed McGuire *et al*. (1957) have published a method of fitting a Poisson binomial distribution which might have application in the case of root borer counts. Two sets of data were fitted to the Poisson binomial. The chief advantage is the ease of fitting. To obtain a reasonable fit a high n value

(8-12) was used. McGuire et al. (1957) state that for high values of n the Poisson binomial rapidly converges to Neyman n = 0, so the latter appeared to be satisfactory.

It has been clearly shown that individual root counts of the clover root borer are not normally distributed and, therefore, that the usual statistical analyses are not valid. However, it is known that for many distributions which are not themselves normal that the distribution of sums, or means of large samples taken from such populations are often normally distributed. Therefore, the distribution of borers in 5 and 10 root samples was investigated.

During a 1954 sampling study 256 sets of 5 roots each were dug. The distribution of the sums of these samples is shown in table 3, as well as the distribution calculated by Neyman's distribution for $n \rightarrow \infty$. Considering the irregularity of the observed counts, the Neyman's distribution gave an excellent fit of these data.

The data just shown can be randomly combined as 128 samples of 10 roots each. These data and the computed Neyman's distribution for $n \longrightarrow \infty$ are shown in table 3.

We can only conclude that the sums, or means of 5 and 10 root samples are not normally distributed, but that they closely agree with Neyman's distribution. Sufficient samples of larger size were unavailable to study accurately other frequency distributions. A few samples that could be constructed of size 20 still appeared to follow the same pattern.

APPLICATION OF THE TRANSFORMATIONS

The mean and variances of samples from 103 fields taken during a State survey in 1954 were equated and a line fitted to linear regression of variance on mean. The samples consisted of about 30 roots each, although a few were as small as 15 or as large as 45. No adjustment was made for the difference in the size of sample in fitting the regression line.

A second degree curve was also fitted to these data and gave a significantly better fit. Figure 1 is a scatter diagram showing these two fitted regression lines. Were this relationship linear, a square root transformation would be suggested. However, the variance became disproportionately large as the mean increased. The data were transformed by $\sqrt{x + 1/2}$. The relationship between transformed mean and variance is shown in figure 2. As expected, the transformation was unsatisfactory, since the variance increased with

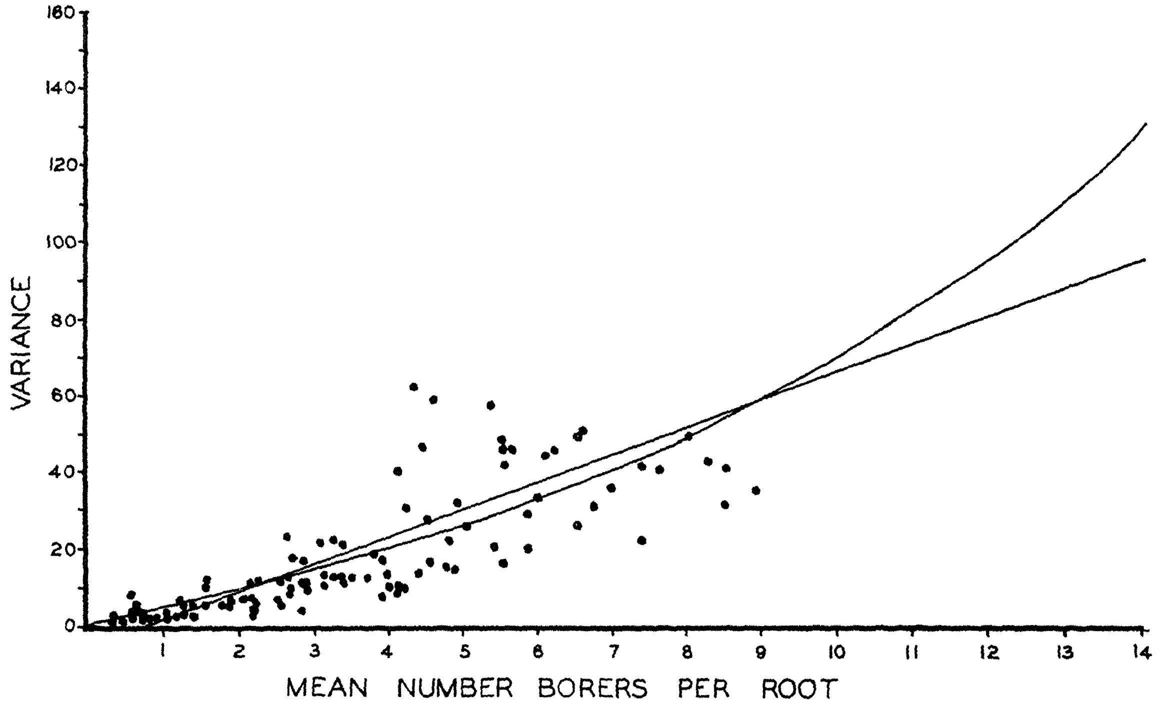

Figure 1. Relationship of mean and variance of non transformed individual root counts showing linear and curvilinear regression lines.

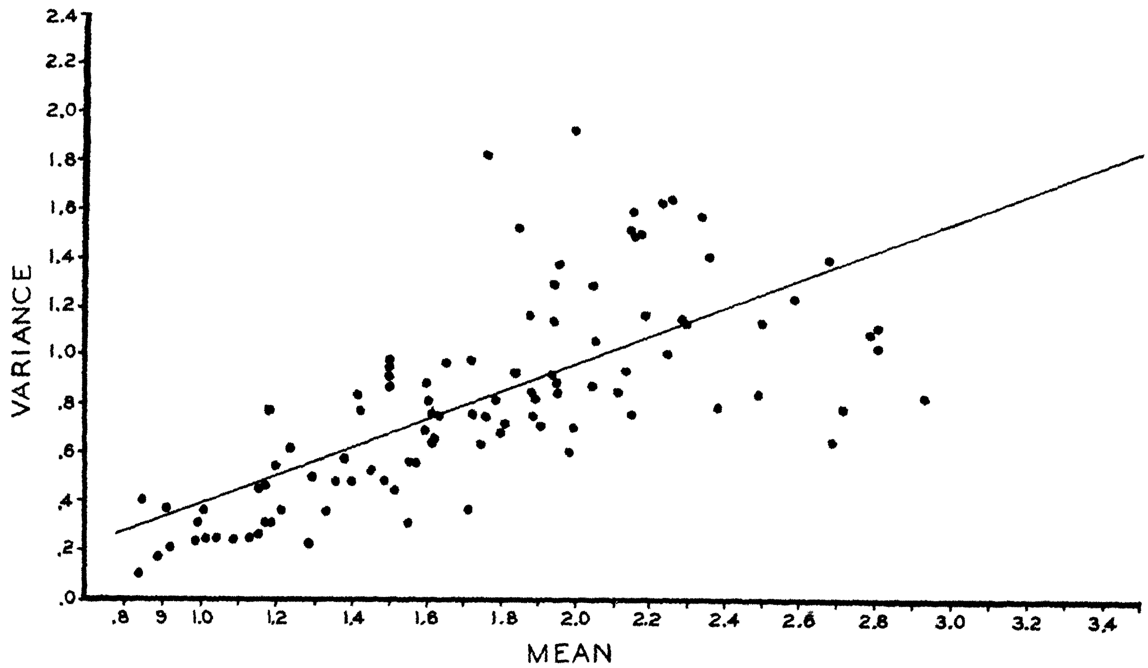

Figure 2. Relationship of mean and variance of individual root counts transformed by $\sqrt{x + 1/2}$.

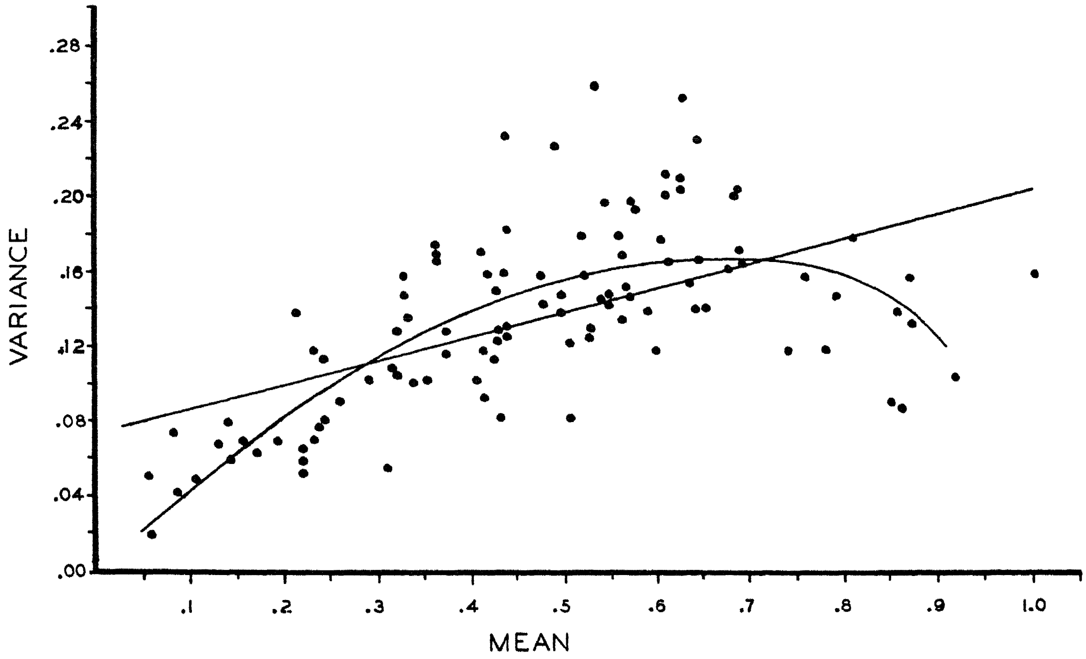

Figure 3. Relationship of mean and variance of individual root counts transformed by log (x + 1).

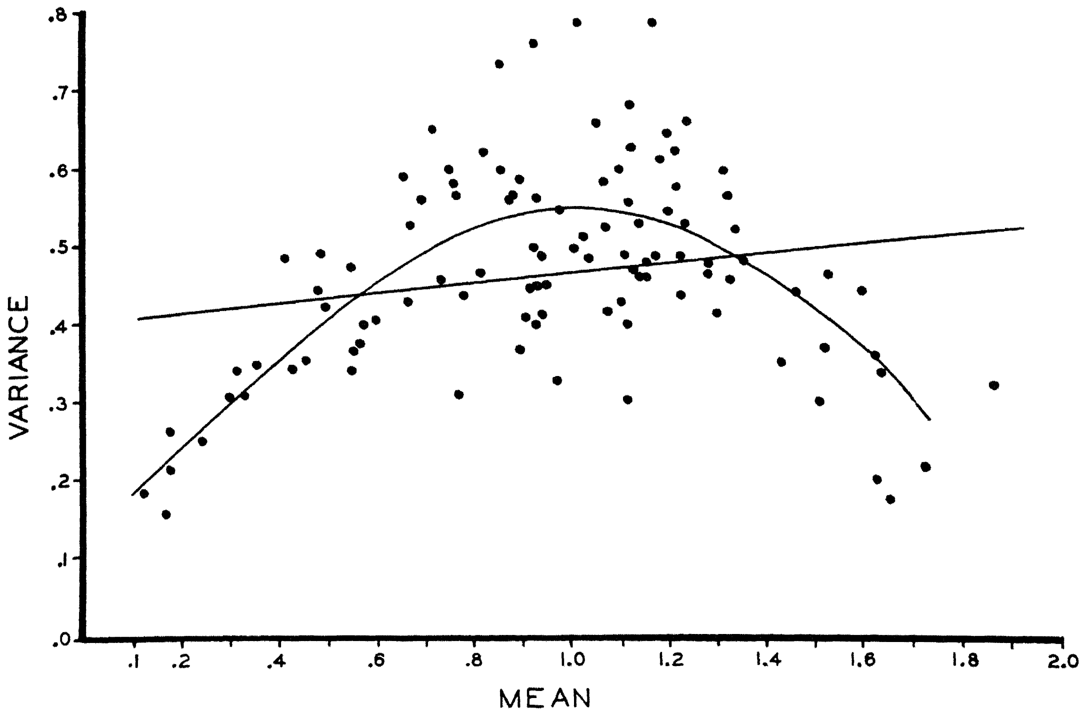

Figure 4. Relationship of mean and variance of individual root counts transformed by inverse hyperbolic sine.

increasing mean.

The authors are not aware of a transformation for counts agreeing with Neyman's distributions; however, log (x + 1) has been used when the variance is roughly proportional to the mean. Accordingly, the counts from these 103 fields were transformed in this way and the mean and the variance plotted as shown in figure 3. A markedly curvilinear relationship resulted, with variance being lowest for extremely low or high populations and highest for medium populations.

Beall (1942) proposed an inverse hyperbolic sine transformation for the negative binomial and Beall and Rescia (1953) suggest that this may be useful for counts agreeing with Neyman's distributions. This transformation is given by $x' = q^{-1/2} \sinh^{-1} (qx)^{1/2}$ and shades from a square root transformation at q = 0 to logarithmic at $q \to \infty$. An estimate of the constant, q, may be obtained from the mean and the variance of the original counts by the relationship of $\mu_2 = \mu_1 + q\mu_1^2$ where $\bar{x}$ and s^2 are used as estimates of μ_1 and μ_2. Beall (1942) gives a good discussion on the estimation of q from field data. He also gives a table for the transformation which is partially reproduced in table 4.

The table has also been extended to include q = 1.5, 2.0, 3.0 4.0 and 5.0. 2/

A common value of q was chosen (q = 1.0) and the data from the 103 fields illustrated in figure 1 transformed. The mean and the variance were again plotted as shown in figure 4. The straight line regression was not significant. However, a second degree curve was highly significant and indicated that variance was highest for samples of intermediate populations. However, since q decreases as the mean increases, the transformation was inexactly applied over the entire range of the mean. Even though inexactly applied, this transformation was effective in reducing and stabilizing variance. In most cases data would not be available over such a wide range of the mean and the transformation should prove quite satisfactory.

For the practical experimenter it would be laborious to calculate a q for each set of data. Since q may be estimated from the mean, the authors have devised a graphical estimate of q for root borer data. Various values of q for transforming individual root counts were calculated from the curved regression line in figure 1 and these data

2/ A. F. Schmitthenner and L. E. Williams, Botany Dept. O.A.E.S., contributed to the extension of the table.

are plotted in figure 5. The lines in figure 5 for 5 and 10 root samples were calculated from the regression lines of figures 6 and 7 respectively.

The regression of variance on means of 5 root counts was computed in the same manner as for individual counts. These data were all obtained from sampling studies conducted in 1955 and 1956. There was a definite increase in variance of non transformed counts as the mean increased, as shown in figure 6. Curvilinearity of regression was not apparent in this case.

Similar computations of the regression of variance on means were made for various transformations. These are illustrated in figure 8, log (X + 1); figure 9, $X^{1/2}$; figure 10, inverse hyperbolic sine; and figure 11, inverse hyperbolic sine applied to individual root counts and combined in sets of five. Obviously, the transformation applied to individual counts is more effective in separating mean and variance.

Similar transformations were made for sums of 10 root samples. Figure 7 shows the relationship between the mean and variance of the original samples; figure 12 shows the same when transformed by log (x + 1); figure 13 for $X^{1/2}$; figure 14 for the inverse hyperbolic sine; figure 15, the inverse hyperbolic sine transformation applied to individual counts and combined in sets of 10.

Note that for samples of this size, these three transformations all tended to reduce and stabilize variance, although scattering of points about the regression line was much greater for the log transformation. Application of the inverse hyperbolic sine transformation to the individual counts showed the least relation between mean and variance.

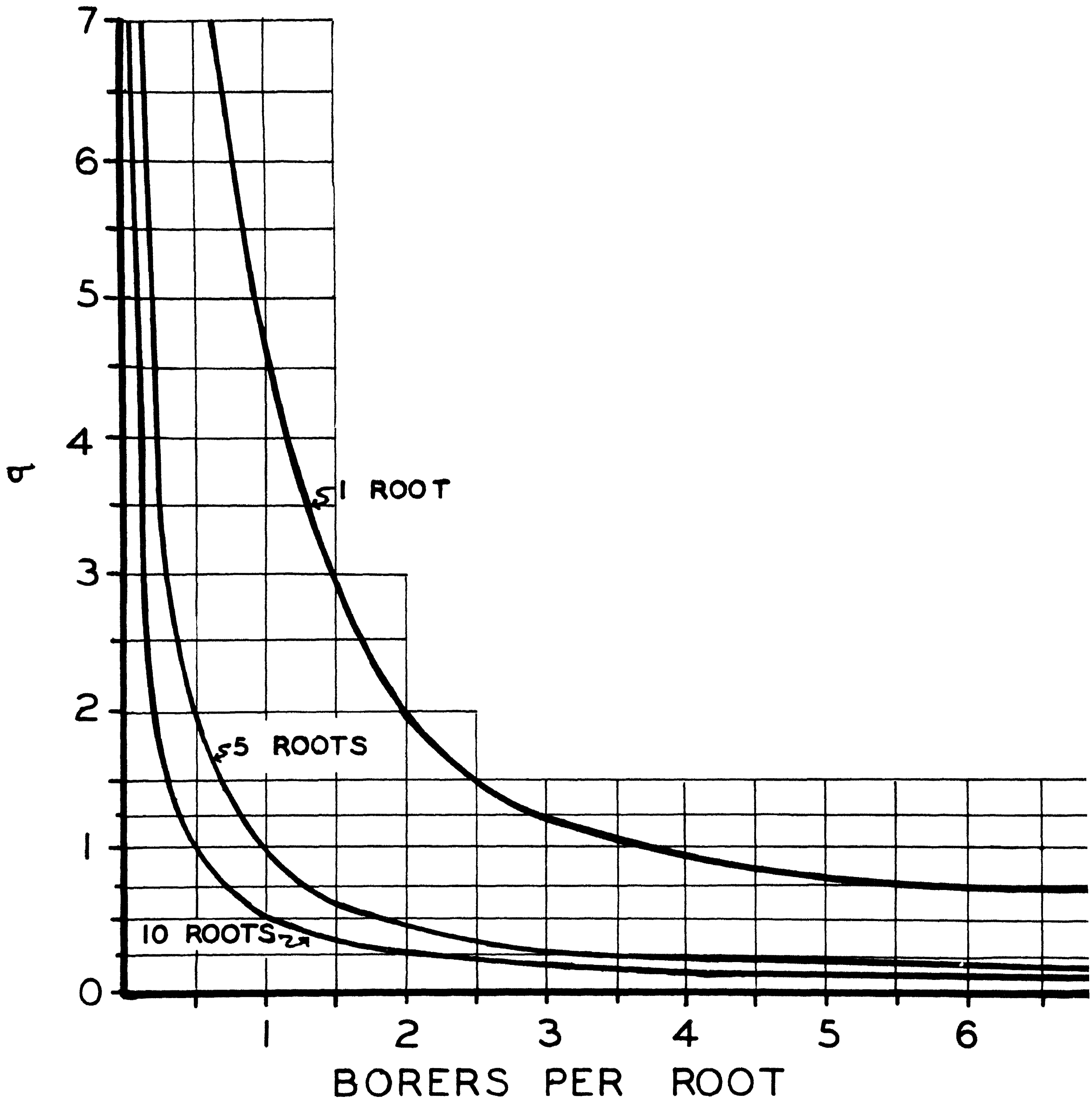

Figure 5. Estimates of q for 1, 5, and 10 root samples for different means for use in inverse hyperbolic sine transformation.

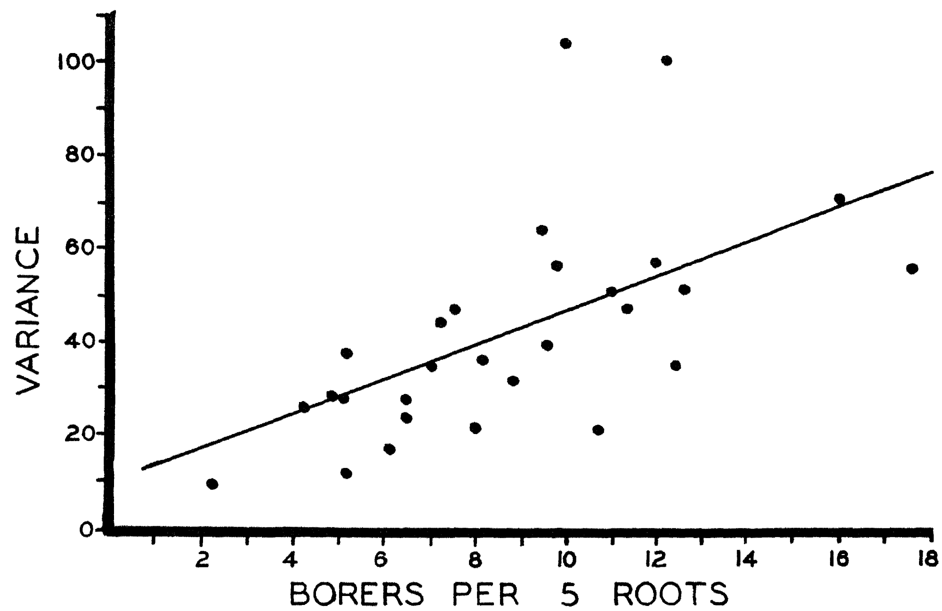

Figure 6. Relationship of mean and variance of non transformed five root samples.

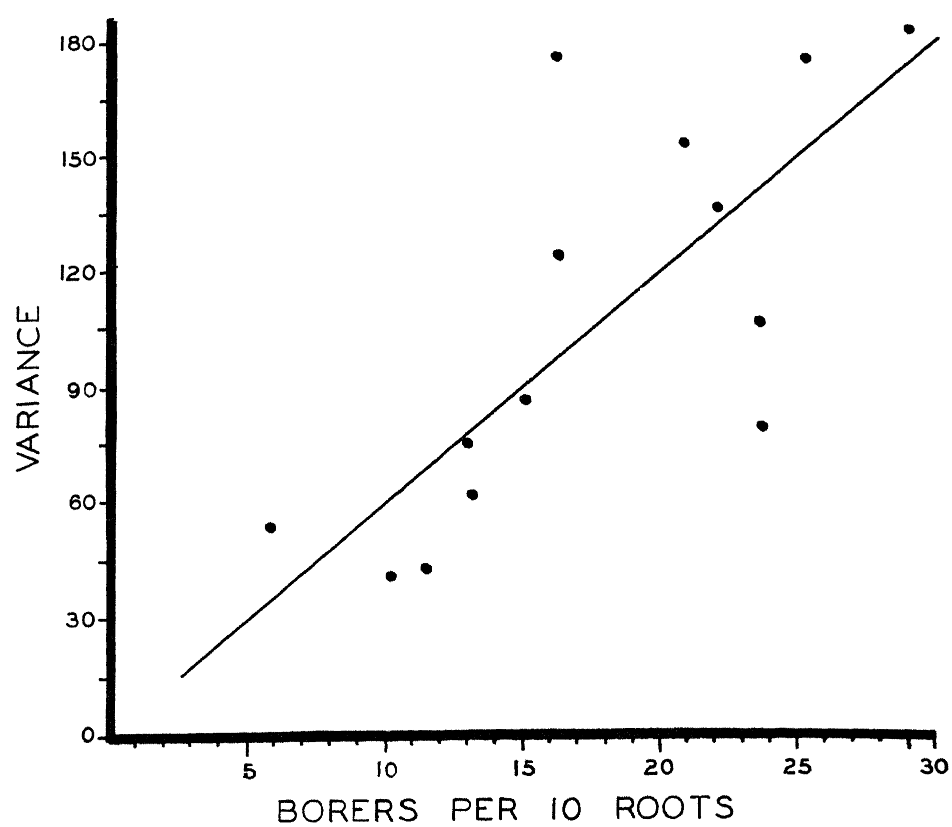

Figure 7. Relationship of mean and variance of non transformed ten root samples.

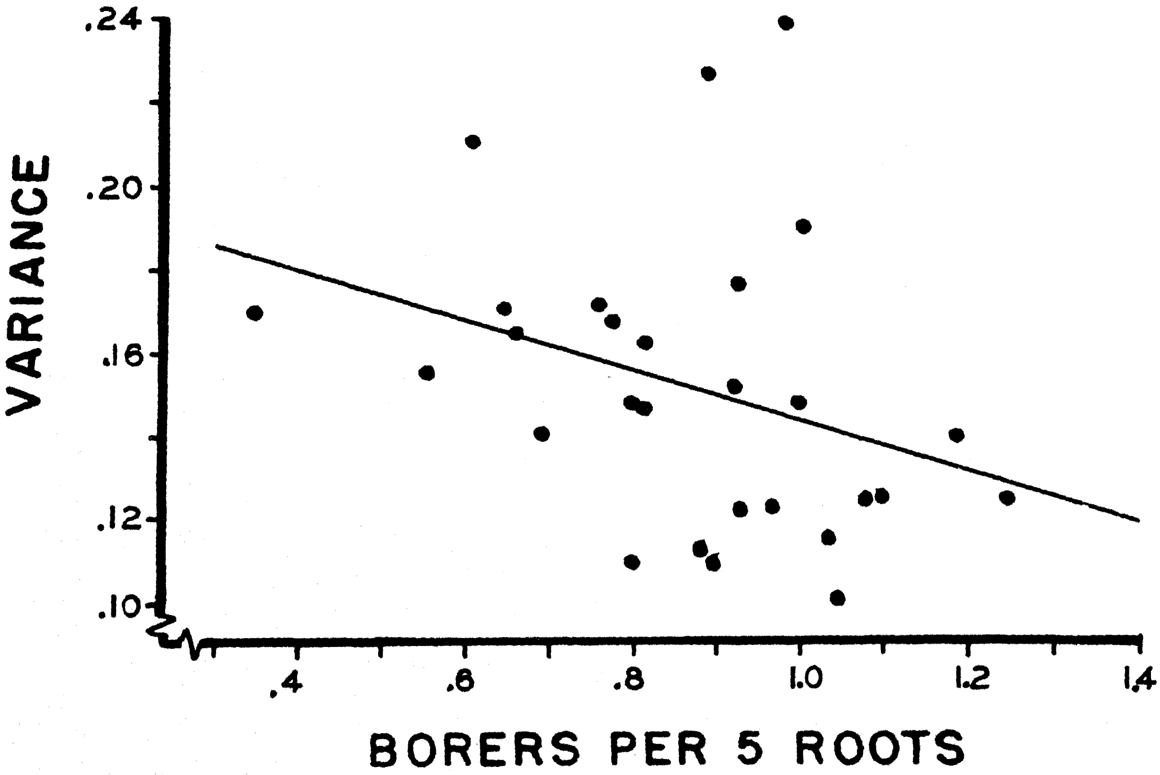

Figure 8. Relationship of mean and variance of five root samples transformed by log (x + 1).

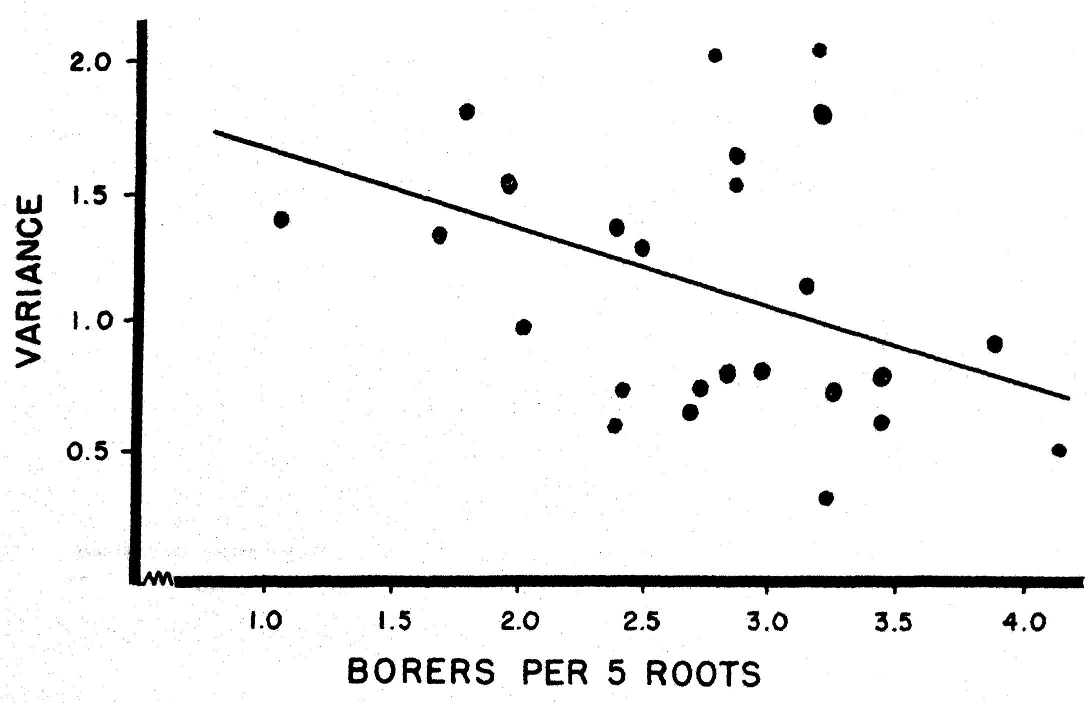

Figure 9. Relationship of mean and variance of five root samples transformed by $\sqrt{x}$

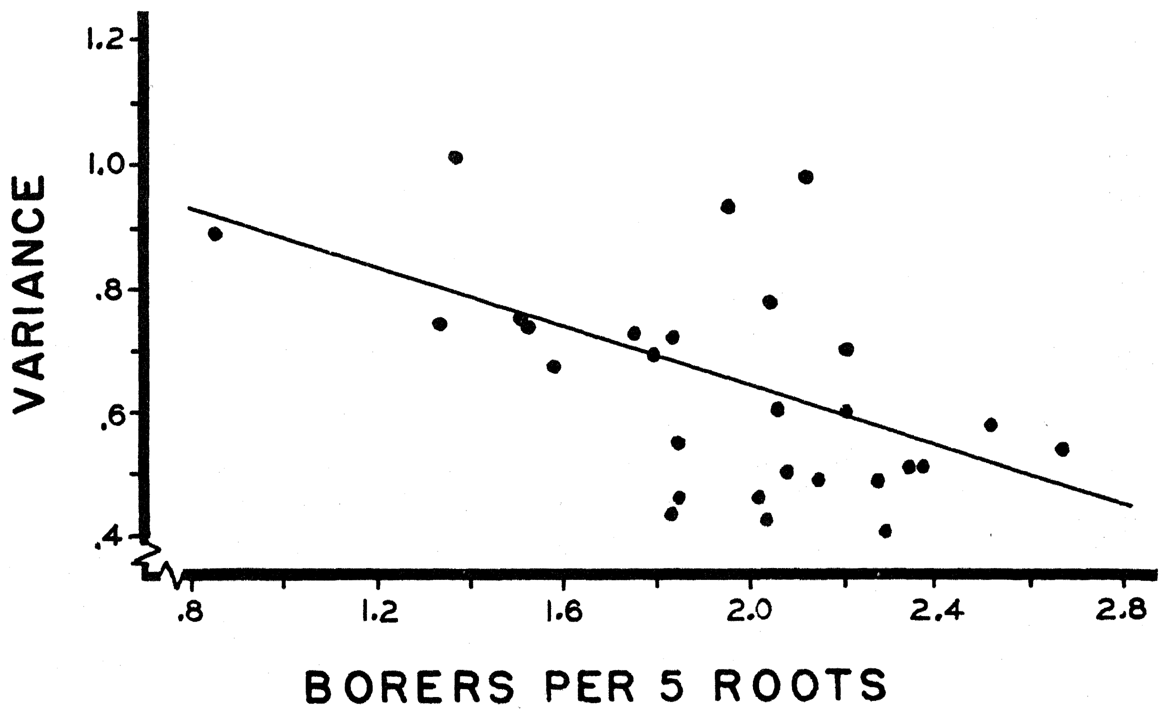

Figure 10. Relationship of mean and variance of five root samples transformed by inverse hyperbolic sine.

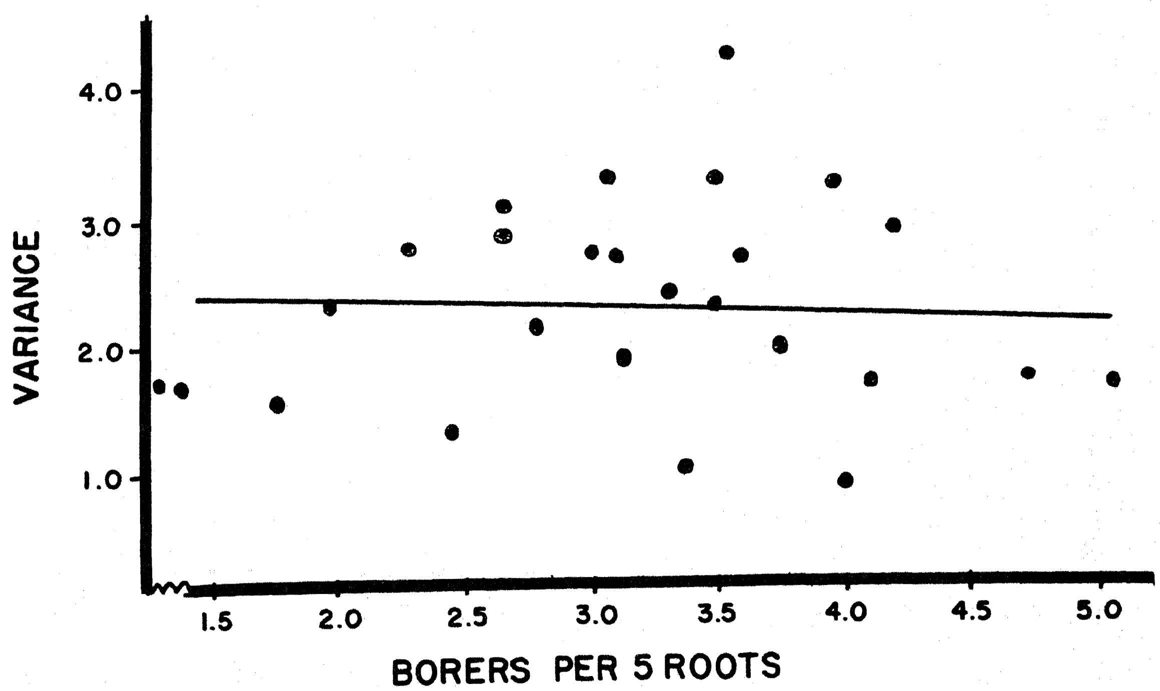

Figure 11. Relationship of mean and variance of individual root samples, transformed by inverse hyperbolic sine, and combined in sets of 5.

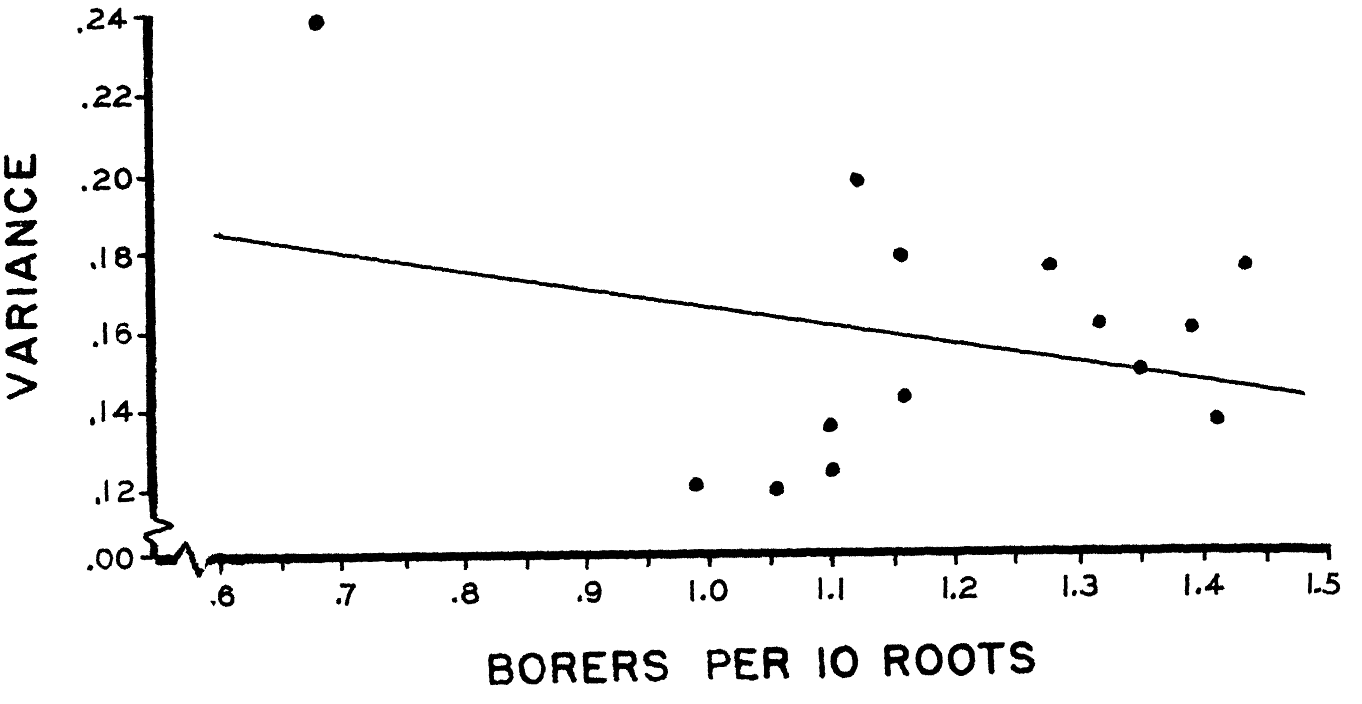

Figure 12. Relationship of mean and variance of ten root samples transformed by log (x + 1).

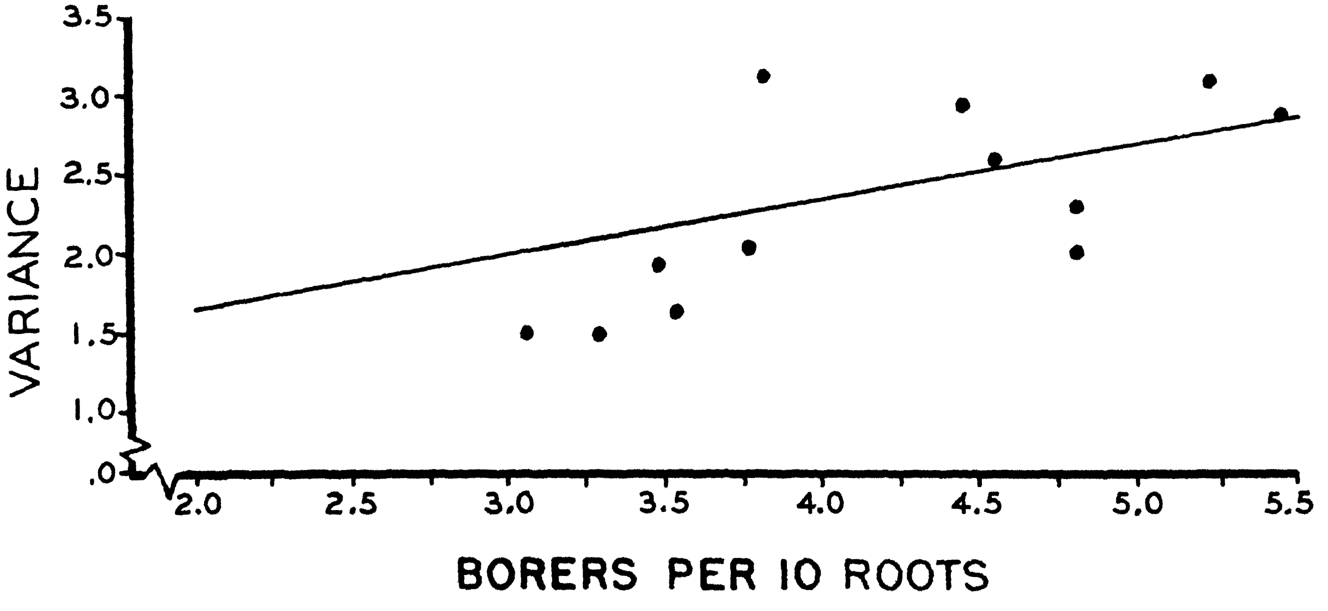

Figure 13. Relationship of mean and variance of ten root samples transformed by $\sqrt{x}$.

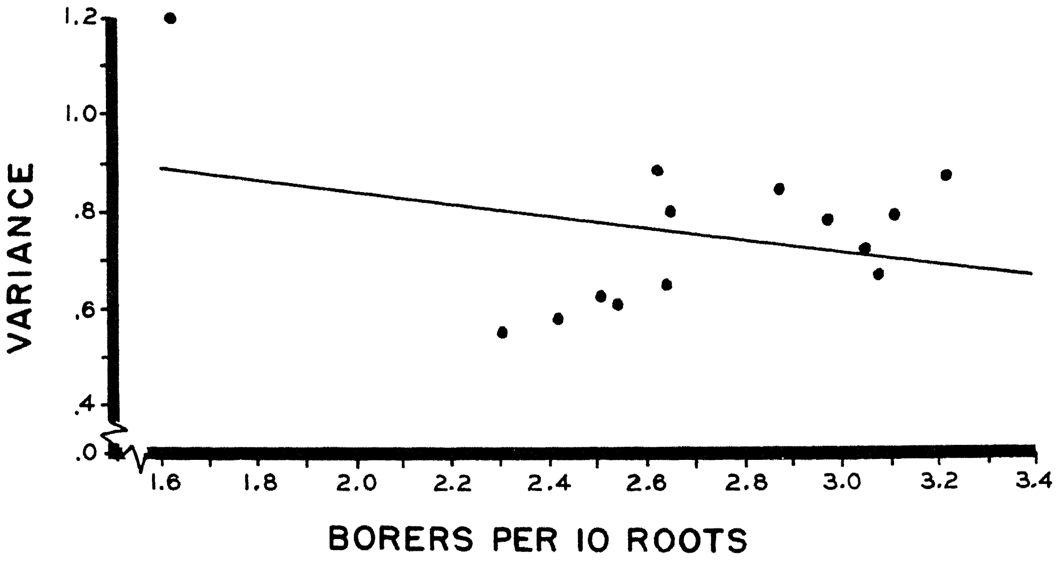

Figure 14. Relationship of mean and variance of ten root samples transformed by inverse hyperbolic sine.

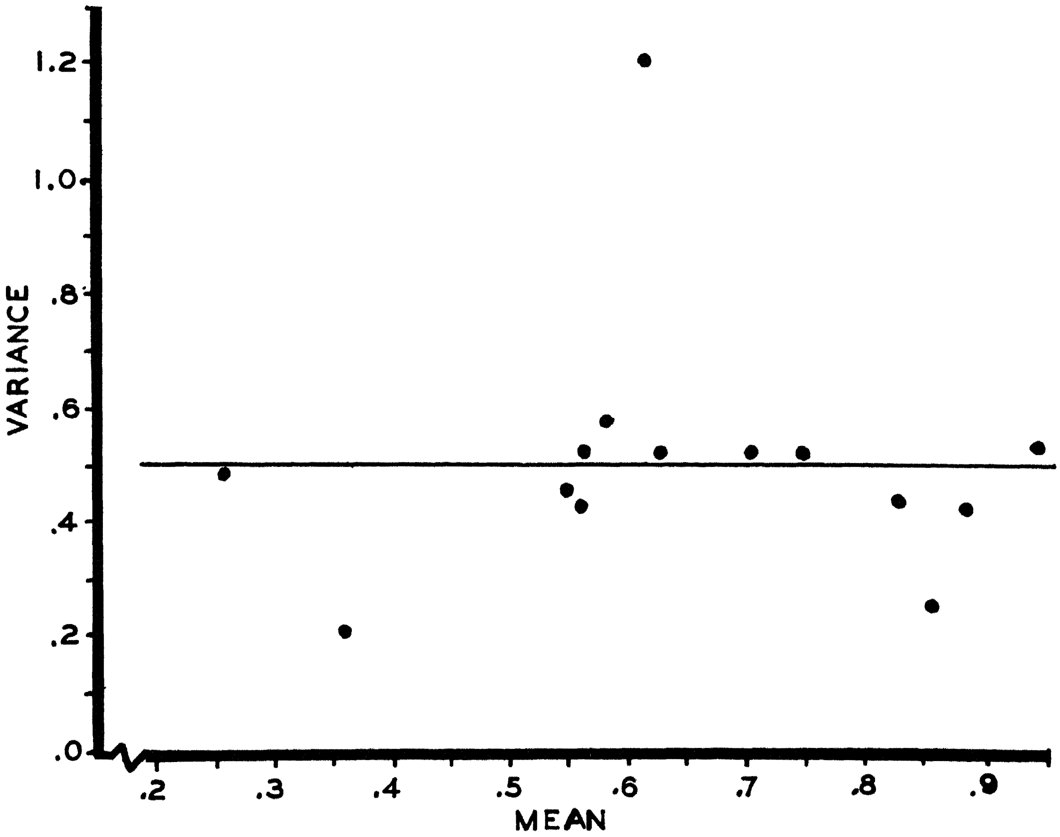

Figure 15. Relationship of mean and variance of individual root samples, transformed by inverse hyperbolic sine, and combined in sets of 10.

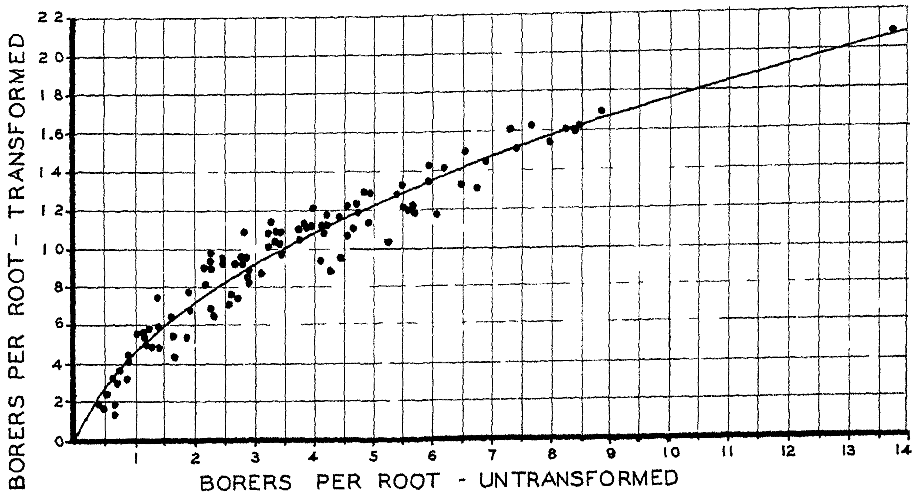

Figure 16. Relationship between transformed and non transformed means for individual root samples.

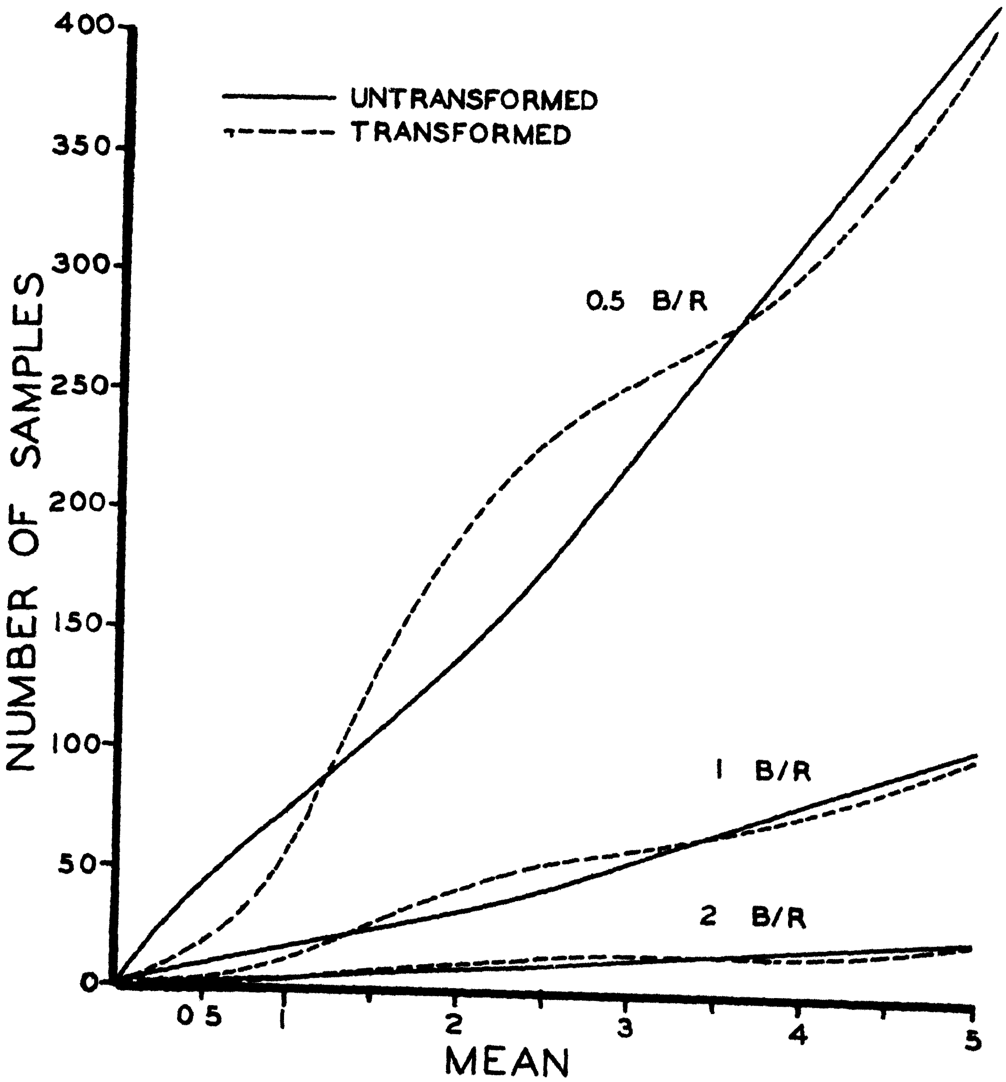

Figure 17. Number of one root samples required to discover specific differences.

CONCLUSIONS ABOUT THE APPLICATION OF TRANSFORMATIONS

The inverse hyperbolic sine transformation appears suitable for root borer counts, either individually or in sets up to ten. Log (x + 1) or $x^{1/2}$ appear equally suitable for sums of 5 or 10 root samples, but both are inferior to the inverse hyperbolic sine for individual root counts. Transformation of individual counts with the inverse hyperbolic sine and combination of the individual transformed counts into sets appears to be the best device for making the mean independent of the variance. However, in field work this may prove to be too laborious for most experimenters. The most practical device seems to be to take the roots in sets and transform the sums of the sets by the square root transformation. However, for the most discriminating experimenter the authors recommend the inverse hyberbolic sine transformation of individual root counts.

EFFECT OF THE TRANSFORMATION ON ADDITIVITY, HOMOGENEITY VARIANCE, AND SIGNIFICANCE TESTS

Several sets of data were available from replicatec field tests that lend themselves to treatment by the analysis of variance. In four such sets the analyses we performed in three ways. The non transformed data, the transformed sum of sets of 10 roots, and the sums of 10 transformed observations were used as variables in each analysis. The analysis of variance lent itself to appli cation of a test for additivity as described by Tukey (1949). The results of the analyses are shown in table

In six of the eight cases the F statistic due to nc additivity was reduced by one of the transformations, although these examples are non conclusive, since only c of four non transformed analyses were significantly non additive. Evidence points to some advantage for the tra formation, however.

Bartlett (1937) devised a test for homogeneity of variance. This test was applied for the four experiment in table 5. The results of the test were compared for transformed and non transformed data and are listed in table 6.

In no case was the Chi square value for homogeneity variance for non transformed counts significant, althouc it was under the 10% level for two of the four experimer In any event, the transformation removed any doubts of r homogeneity in all cases.

A desirable characteristic of a transformation woul be the rendition of the scale of measure into one that showed the highest significance in tests of hypothesis. There may occur cases in which the transformed scale was a more suitable one than the original and other cases where the transformed scale is less suitable. A few cas of field experiments were available in which the test statistics computed from transformed data could be compa with statistics from the original data.

Two sets of data that did not lend themselves to Tukey's test were analyzed in the usual way using non transformed sums of ten roots and sums of ten transforme individual observations. The results are shown in table

In these tests and those shown in table 5 five of five F values computed from transformations of individual counts were rendered more significant. In the sixth case the treatment interaction was rendered non significant. This latter case might be considered an advantage since the data has now been scaled to lead to simple interpretation of "A" effects without the complication of considering "B".

Data were available from eight experiments in which populations under one condition were compared with populations under some other condition. These data led to the use of a t statistic for comparing the two population means. t tests were performed using non transformed and transformed data. A comparison of the t statistic using the two variables is made in table 8. There is a tendency for the transformed counts to show a higher t value than the non transformed values. There are two exceptions in the eight tests.

POWER OF TRANSFORMED COUNTS

In addition to divorcing mean and variance and improving additivity a transformation should not lose power, the ability to discover differences. In order to test the power of the two types of variables, transformed and non transformed computations were made comparing the number of samples necessary to discover a specific difference from a true mean. This type of test is not the most commonly employed in experimental work but will be sufficient to serve as a guide in judging power.

Alpha, or the risk of rejecting a sample like the true mean, was set at .05. Beta, or the risk of accepting a sample unlike the true mean, was set at .10. The normal deviate was used as a test criterion to compute the number of samples necessary to discover specific differences from a true mean.

For the original non transformed data, the variance increased as the mean increased (figure 1). Therefore, the size of sample to discover a specific difference would be expected to vary over the range of the mean. If, for various mean population levels, one substitutes the appropriate standard deviation, the resulting calculations are those shown in table 9. It is apparent that it requires a much larger sample to discover a specific difference at a high population than at a low one, due to the rapid increase in variance over the mean.

One can also perform a similar type of analysis for the transformed counts. The only new problem arising here is that a unit change in borer per root is not a uniform change in the transformed variable, but changes as the mean changes. This change can be calculated from the function involved in transforming the original counts to the transformed. Figure 16 is a plot of the function and the graph was used to make the calculation. Again, the variance is not entirely independent of the mean, but can be estimated from figure 4.

After the adjustments were made, the calculation of sample sizes to discover specific differences in terms of borers per root is given in table 10. The difference to be discovered must also be expressed in terms of the transformed variable. This also changes with the mean. For example, a difference of 1.0 borers per root expressed in terms of the new variable changes from 0.35 at a mean of 0.5 borers per root to 0.08 at a mean of 8.0 borers per root.

Figure 17 compares the number of root samples required to discover differences of 0.5, 1.0, and 2.0 borers per root for both transformed and non transformed counts. The two curves closely parallel one another in all cases, although there is a slight tendency for the transformation to be more efficient at low population means and to lose power at high means.

From these calculations one can conclude that the transformation does not lose power in discovering difference from a true mean.

EFFICIENCY OF SAMPLING DESIGNS

We wish that any sample that we take give an efficient estimate of the population. Sampling clover root borers may have two objectives: evaluating experimental treatments and surveying a field or area. For either of these objectives, we wish to choose an experimental design which gives us the most reliable estimate, but minimizes sampling cost. Two sampling experiments were conducted to give information about the most suitable sampling plan for either of the two objectives of the experimenter.

In 1955 a single field was divided into four ranges, one range being taken from each corner of the field. Each range was further subdivided into four blocks and each block into four plots. Four sets of five roots each were dug in each plot. The plots measured 20 by 50 feet. In 1956 three fields were sampled. Two ranges were selected from each field and divided into two blocks of four plots each. As in the 1955 study the plots were 20 by 50 feet and four sets of five roots each were dug from each plot. The individual root counts were transformed by the inverse hyperbolic sine ($q = 1.5$) and an analysis of variance performed on the data to evaluate the components of variance attributed to the elements of the sampling plan. Tables 11 and 12 give the analyses. With the values for the components serving as estimates, we can compute an estimated variance for 1) a survey of an area of several fields or 2) the sampling variance within a plot in an experiment. For either objective in sampling the precision will rise as the larger subdivisions were sampled more extensively. This is naturally true; however, sampling the larger subdivisions (ranges and fields) usually adds to the cost of the sampling at a greater rate than does sampling the small units. In the case of clover roots, several can be dug in one set in the same time that it would take to locate another range or another field. Accordingly, cost functions were assigned to the various sampling subdivisions. These cost functions are arbitrary and have no basis for their values other than the judgment of the authors who have had some experience in sampling this insect. If the cost (time, money, and effort) of digging one root was assigned the value of one, other costs would be as follows: set, 1.1; plot, 1.5; block, 2.0; range, 10.0 and field 50.0. Using these cost functions, the cost of a particular sampling plan could be assessed. If the cost of a sampling plan can be fixed and there are available estimates of the cost functions and the components of variance, one may compute the sampling plan that will minimize the variance. Consider the following quantities: C_1 = cost of sampling a single unit: σ_1^2 =the variance component ascribed to the sampling unit : C_o = the total cost of the sampling plan; n_i = the nunber of sub units in any larger unit; the variance of the sample =

$$\frac{\sigma^2 k}{n_i \ldots n_k} + \frac{\sigma^2 k-1}{n_i \ldots n_{k-1}} + \ldots\ldots + \frac{\sigma_2^2}{n_1 \quad n_2} + \frac{\sigma_1^2}{n_1}$$

The number of various sampling units to take in order to minimize variance under the restriction of a fixed cost will be given by solving for the following n's. 3/

$$n_1 = \frac{\sigma_1}{\sqrt{C_1}} \cdot \frac{C_o}{\sum_j \sigma_j \sqrt{C_j}}$$

$$n_k = \frac{\sigma_k}{\sigma_{k-1}} \cdot \frac{\sqrt{C_{k-1}}}{\sqrt{C_k}}$$

For a comparable example, see Bancroft and Brindley (1956).

Sampling plans that will give minimum variance have been computed, using the above procedure. In addition,

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3/ The assistance of D. R. Whitney, Dept. of Mathematics, Ohio State University in deriving the formula is gratefully acknowledged.

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the cost and variance have been computed for the sampling plan used, as well as some alternate plans that an experimenter might consider.

The variance of an alternate plan has been divided by the variance of the plan used in this study. This ratio is expressed as "relative precision". (Usually in statistical literature this is "relative efficiency" but the latter term will be reserved for another concept to be shown later). The cost of an alternate plan has been divided by the cost of the plan used in the study and this ratio termed "relative cost". Finally, the ratio, relative precision, has been divided into the ratio, relative cost. This third ratio of precision to cost is termed "relative efficiency". It serves as a measure of the variation in terms of cost.

If an experimenter wishes to sample a group of experimental plots, he would like to use the most efficient sampling plan within the plot. The sampling variance in this instance would be equal to

$$\frac{\sigma_r^2}{rs} + \frac{\sigma_s^2}{s} \quad \text{where} \quad \sigma_r^2 \quad \text{and} \quad \sigma_s^2 \quad \text{are}$$

the components of variance due to roots and sets of roots. r and s are the number of roots per set and the sets per plot, respectively. Both the 1955 and 1956 experiments give estimates of these components and the variance, cost and relative efficiency of some alternate plans are shown in table 13. Apparently, the most efficient design of sampling within a plot is to take 3 roots in each of 6 sets.

The above computations apply to sampling of field plots such as one might encounter in tests of experimental material. Another problem involves the surveying of populations in an area composed of several fields. The above technique was applied to this problem. Only the 1956 data were used, since the 1955 sampling gave no estimate of field to field variation. Several plans are represented in table 14. Fourteen (16) fields, composed of 6 (6) blocks of 1 (1) plot of 2 (2) sets of 3 (3) roots, were the two most efficient plans. This should serve as a guide for area surveyors.

SUMMARY

This paper deals with counts of the number of clover root borers in red clover roots. It has attempted to make available to workers in insect control and survey efforts a technique which will improve the sampling designs and the analysis of the data. Several sample distributions were fitted to compound Poisson type distributions and Neyman's distributions were found to give a reasonable fit. Several transformations of the data were applied and a transformation based on the inverse hyperbolic sine was found to be most satisfactory in stabilizing variance. Beall's table of the inverse hyperbolic sine is extended to include data that are likely to arise in root borer sampling. Because a paramenter, q, is necessary in the transformation, values of q have been estimated and tabulated graphically for clover root borer samples.

The transformation appeared to increase significance of various conventional test statistics; it reduced non-additivity; divorced the mean and variance; and did not effect power.

The efficiency of several sampling plans was investigated and suggestions presented for best sampling techniques for 1) area sampling 2) experimental plot sampling.

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SAMPLING STUDIES
OF THE
CLOVER ROOT BORER TABLES

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Table I. Summary of frequency distributions fitted to clover root borer data, Virginia, Experiment I, 1954 (treated plots).

Borers Per Root	Observed Frequency	Calculated Frequency					
		Negative	Neyman			Double Poisson	
		Binomial	n = 0	n = 1	$n \to \infty$	Method 1	Method 2
0	199	165.1	208.8	198.9	183.1	211.6	199.0
1	12	48.8	5.5	15.8	29.9	2.8)	12.0
2	17	26.8	11.1	15.8	22.5	9.1)	23.9
3	23	17.0	15.2	15.0	16.8	14.7	23.5
4	10	11.5	15.7	13.4	12.5	16.4	17.1
5	11	8.0	13.4	11.2	9.3	14.0	10.3
6	6	5.8	9.9	8.8	6.9	10.0	6.1
7	7	4.2	6.7	6.5	5.1	6.6	3.7)
8	5	3.1	4.5	4.6	3.8	4.3	2.2)
9+	10	9.7	9.2	10.0	10.1	10.5	2.2)
	300	300.0	300.0	300.0	300.0	300.0	300.0
χ^2		44.78	19.46	7.09	17.75	34.90	29.28
P_{χ^2}		< 0.01	< 0.01	0.22	< 0.01	< 0.01	< 0.01

Table 2. Frequency of occurrence of probabilities of chi square computed from 23 sample distributions of clover root borer counts.

Probability	Frequency of Occurrence in Negative Binomial	Frequency of Occurrence in Neyman's
0 - .09	12	6
.10 - .19	4	3
.20 - .29	2	0
.30 - .39	0	2
.40 - .49	2	3
.50 - .59	0	2
.60 - .69	1	2
.70 - .79	0	1
.80 - .89	2	1
.90 - 1.00	0	3
Total	23	23

Table 3. Observed distributions of borers in 5 and 10 root samples and distribution calculated from Neyman's $n \to \infty$. Wooster, Ohio, 1954.

Borers in 5 Roots	Observed	Calculated	Borers in 10 Roots	Observed	Calculated
0	5	4.1	0	0	0.1
1	6	6.4	1	0	0.2
2	10	8.9	2	1	0.4
3	13	11.2	3	0	0.7
4	12	12.8	4	2	1.0
5	17	14.5	5	1	1.4
6	15	15.5	6	2	1.7
7	16	15.9	7	4	2.2
8	9	16.0	8	5	2.6
9	15	15.7	9	3	3.0
10	9	15.1	10	1	3.4
11	22	14.3	11	4	3.7
12	8	13.3	12	2	4.0
13	24	12.2	13	7	4.3
14	10	11.0	14	5	4.6
15	8	9.9	15	2	4.7
16	5	8.8	16	6	4.9
17	5	7.7	17	5	5.0
18	7	6.7	18	3	5.0
19	5	5.8	19	2	5.0
20	5	5.0	20	3	4.9
21	8	4.3	21	2	4.8
22	3	3.6	22	7	4.7
23	3	3.0	23	4	4.5
24+	16	14.3	24	5	4.3
	256.0	256.0	25	5	4.1
			26	7	3.9
			27	4	3.6
			28	4	3.4
			29	6	3.2
			30	2	2.9
			31+	24	25.8
				128.0	128.0

$\chi^2 = 28.61 \quad P_{\chi^2} = 0.08$ (5 roots)

$\chi^2 = 12.78 \quad P_{\chi^2} = 0.37$ (10 roots)

Table 4.

Values of the inverse hyperbolic sine transformation, $x' = q^{-1/2} \sinh^{-1} (qx)^{1/2}$.

X	q							
	0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	1.04	1.00	0.98	0.97	0.96	0.94	0.93	0.92
2	1.40	1.41	1.37	1.33	1.30	1.27	1.25	1.22
3	1.74	1.69	1.66	1.59	1.54	1.50	1.46	1.42
4	1.99	1.94	1.89	1.80	1.73	1.67	1.62	1.58
5	2.23	2.16	2.08	1.97	1.88	1.81	1.75	1.70
6	2.47	2.35	2.25	2.12	2.01	1.93	1.86	1.80
7	2.62	2.50	2.41	2.25	2.13	2.04	1.96	1.89
8	2.78	2.68	2.54	2.36	2.23	2.13	2.04	1.97
9	2.95	2.81	2.67	2.47	2.32	2.21	2.12	2.04
10	3.11	2.95	2.79	2.56	2.40	2.28	2.18	2.10
11	3.26	3.07	2.89	2.65	2.48	2.35	2.25	2.16
12	3.39	3.19	3.00	2.73	2.55	2.41	2.30	2.21
13	3.53	3.30	3.09	2.81	2.62	2.47	2.36	2.26
14	3.66	3.41	3.18	2.88	2.68	2.52	2.40	2.31
15	3.78	3.51	3.26	2.94	2.73	2.57	2.45	2.35
16	3.90	3.61	3.34	3.01	2.79	2.62	2.49	2.39
17	4.01	3.69	3.42	3.07	2.84	2.67	2.53	2.42
18	4.12	3.79	3.49	3.12	2.88	2.71	2.57	2.46
19	4.23	3.87	3.56	3.18	2.93	2.75	2.61	2.49
20	4.33	3.94	3.62	3.23	2.97	2.79	2.64	2.52
21	4.43	4.03	3.69	3.28	3.01	2.82	2.68	2.56
22	4.53	4.10	3.75	3.32	3.05	2.86	2.71	2.58
23	4.63	4.17	3.81	3.37	3.09	2.89	2.74	2.61
24	4.72	4.24	3.86	3.41	3.13	2.92	2.77	2.64
25	4.81	4.31	3.92	3.45	3.16	2.95	2.79	2.66
26	4.89	4.38	3.97	3.49	3.20	2.98	2.82	2.69
27	4.98	4.44	4.02	3.53	3.23	3.01	2.85	2.71
28	5.06	4.50	4.07	3.57	3.26	3.04	2.87	2.73
29	5.15	4.56	4.12	3.61	3.29	3.07	2.89	2.76
30	5.23	4.61	4.16	3.64	3.32	3.09	2.92	2.78
31	5.31	4.68	4.21	3.67	3.35	3.12	2.94	2.80
32	5.39	4.73	4.25	3.71	3.38	3.14	2.96	2.82
33	5.47	4.79	4.30	3.74	3.40	3.16	2.98	2.84
34	5.54	4.84	4.34	3.77	3.43	3.19	3.00	2.86

Table 4 - continued

Values of the inverse hyperbolic sine transformation, $x' = q^{-1/2} \sinh^{-1} (qx)^{1/2}$

X	q 0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60
35	5.61	4.89	4.38	3.80	3.45	3.21	3.02	2.88
36	5.68	4.94	4.42	3.83	3.48	3.23	3.04	2.89
37	5.75	4.99	4.46	3.86	3.50	3.25	3.06	2.91
38	5.82	5.04	4.49	3.89	3.53	3.27	3.08	2.93
39	5.89	5.08	4.53	3.91	3.55	3.29	3.10	2.94
40	5.96	5.12	4.57	3.94	3.57	3.31	3.12	2.96
41	6.04	5.17	4.60	3.97	3.59	3.33	3.13	2.98
42	6.09	5.20	4.63	3.99	3.61	3.35	3.15	2.99
43	6.15	5.24	4.67	4.02	3.63	3.37	3.17	3.01
44	6.21	5.29	4.70	4.04	3.65	3.39	3.18	3.02
45	6.31	5.33	4.73	4.07	3.67	3.40	3.20	3.03
46	6.37	5.37	4.76	4.09	3.69	3.42	3.21	3.05
47	6.41	5.41	4.80	4.11	3.71	3.44	3.23	3.06
48	6.47	5.45	4.83	4.13	3.73	3.45	3.24	3.08
49	6.52	5.49	4.85	4.16	3.75	3.47	3.26	3.09
50	6.58	5.52	4.88	4.18	3.77	3.48	3.27	3.10
55	6.93	5.70	5.02	4.28	3.85	3.56	3.34	3.16
60	7.13	5.87	5.15	4.37	3.93	3.62	3.40	3.22
65	7.37	6.03	5.27	4.46	4.00	3.69	3.45	3.27
70	7.61	6.17	5.38	4.54	4.07	3.74	3.50	3.32
75	7.84	6.32	5.48	4.61	4.13	3.80	3.55	3.36
80	8.06	6.44	5.57	4.68	4.19	3.85	3.60	3.40
85	8.24	6.56	5.66	4.75	4.24	3.90	3.64	3.44
90	8.46	6.68	5.75	4.81	4.29	3.94	3.68	3.48
95	8.63	6.79	5.83	4.87	4.34	3.98	3.72	3.51
100	8.80	6.89	5.91	4.93	4.39	4.02	3.75	3.54
110	9.16	7.09	6.05	5.03	4.47	4.10	3.82	3.60
120	9.48	7.30	6.18	5.13	4.55	4.16	3.88	3.66
130	9.78	7.43	6.31	5.21	4.62	4.23	3.94	3.71
140	10.04	7.58	6.42	5.30	4.69	4.29	3.99	3.76
150	10.26	7.72	6.53	5.37	4.75	4.34	4.04	3.80

Table 4 - continued

Values of the inverse hyperbolic sine transformation, $x' = q^{-1/2} \sinh^{-1} (qx)^{1/2}$

X	q 0.70	0.80	1.00	1.50	2.00	3.00	4.00	5.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.91	0.90	0.88	0.84	0.81	0.76	0.74	0.69
2	1.20	1.18	1.15	1.08	1.02	0.94	0.88	0.84
3	1.40	1.37	1.32	1.22	1.15	1.05	0.93	0.92
4	1.54	1.50	1.44	1.33	1.25	1.13	1.05	0.98
5	1.66	1.61	1.54	1.41	1.32	1.19	1.10	1.03
6	1.76	1.71	1.63	1.49	1.38	1.24	1.14	1.07
7	1.84	1.78	1.70	1.54	1.43	1.29	1.16	1.11
8	1.91	1.85	1.76	1.60	1.48	1.32	1.22	1.14
9	1.98	1.92	1.82	1.64	1.52	1.36	1.24	1.16
10	2.04	1.97	1.87	1.68	1.56	1.39	1.27	1.19
11	2.09	2.02	1.91	1.72	1.59	1.41	1.30	1.21
12	2.14	2.07	1.96	1.76	1.62	1.44	1.32	1.23
13	2.19	2.11	1.99	1.79	1.65	1.46	1.34	1.24
14	2.23	2.15	2.03	1.82	1.67	1.48	1.36	1.26
15	2.27	2.19	2.06	1.85	1.70	1.50	1.37	1.28
16	2.31	2.22	2.09	1.87	1.72	1.52	1.39	1.29
17	2.34	2.25	2.12	1.89	1.73	1.54	1.40	1.30
18	2.37	2.28	2.15	1.92	1.76	1.55	1.42	1.32
19	2.40	2.31	2.18	1.94	1.78	1.57	1.43	1.33
20	2.43	2.34	2.20	1.96	1.80	1.59	1.44	1.34
21	2.46	2.37	2.23	1.98	1.81	1.60	1.46	1.35
22	2.49	2.39	2.25	2.00	1.83	1.61	1.47	1.36
23	2.51	2.42	2.27	2.02	1.85	1.62	1.48	1.37
24	2.54	2.44	2.29	2.03	1.86	1.64	1.49	1.38
25	2.56	2.46	2.31	2.05	1.88	1.64	1.50	1.39
26	2.59	2.48	2.33	2.06	1.89	1.66	1.51	1.40
27	2.61	2.51	2.35	2.08	1.90	1.67	1.52	1.41
28	2.63	2.53	2.37	2.09	1.92	1.68	1.53	1.41
29	2.65	2.54	2.39	2.11	1.93	1.69	1.54	1.42
30	2.67	2.56	2.40	2.12	1.94	1.70	1.54	1.43
31	2.69	2.58	2.42	2.14	1.95	1.71	1.55	1.44
32	2.71	2.60	2.43	2.15	1.96	1.72	1.56	1.44
33	2.73	2.62	2.45	2.16	1.97	1.73	1.57	1.45
34	2.75	2.63	2.46	2.17	1.98	1.74	1.58	1.46

Table 4 - continued

Values of the inverse hyperbolic sine transformation $x' = q^{-1/2} \sinh^{-1}(qx)$

X	q 0.70	0.80	1.00	1.50	2.00	3.00	4.00	5.00
35	2.77	2.65	2.48	2.19	1.99	1.74	1.58	1.46
36	2.78	2.66	2.49	2.20	2.00	1.75	1.59	1.47
37	2.80	2.68	2.51	2.21	2.01	1.76	1.60	1.48
38	2.81	2.69	2.52	2.22	2.02	1.77	1.60	1.48
39	2.83	2.71	2.53	2.23	2.03	1.77	1.61	1.49
40	2.84	2.72	2.54	2.24	2.04	1.78	1.62	1.49
41	2.86	2.73	2.56					
42	2.87	2.75	2.57					
43	2.89	2.76	2.58					
44	2.90	2.77	2.59					
45	2.91	2.79	2.60					
46	2.93	2.80	2.61					
47	2.94	2.81	2.62					
48	2.95	2.82	2.63					
49	2.96	2.83	2.64					
50	2.97	2.84	2.65					
55	3.03	2.90	2.70					
60	3.08	2.94	2.74					
65	3.13	2.99	2.78					
70	3.18	3.03	2.82					
75	3.21	3.07	2.86					
80	3.25	3.10	2.89					
85	3.29	3.14	2.92					
90	3.32	3.17	2.95					
95	3.35	3.20	2.97					
100	3.38	3.23	3.00					
110	3.44	3.28	3.05					
120	3.50	3.33	3.09					
130	3.54	3.37	3.13					
140	3.59	3.42	3.17					
150	3.63	3.45	3.20					

Table 5.

Source of Variation	df	F Values For: Non Transformed	Transformed Sums	Sums of Individuals Transformed
Experiment I				
Total	39	--	--	--
Reps	9	1.91	0.87	2.17
Treatment	3	3.05	1.87	3.19*
Non Additivity	1	11.69**	2.22	10.86**
Error M. S.	26	7.31	0.37	0.51
Experiment II				
Total	17	--	--	--
Reps	2	0.35	0.23	0.34
Treatment	5	7.35**	6.54**	13.57**
Non Additivity	1	0.81	3.17	0.07
Error M. S.	9	34.4	0.52	1.82
Experiment III				
Total	39	--	--	--
Reps	9	0.76	1.14	1.51
Treatment	3	1.07	2.93	1.21
Non Additivity	1	1.13	2.24	0.11
Error M. S.	26	24.01	0.44	1.04
Experiment IV				
Total	14	--	--	--
Reps	2	0.41	0.19	0.63
Treatment	4	8.58**	1.81	14.11**
Non Additivity	1	0.26	0.12	0.12
Error M. S.	7	27.57	0.95	1.60

Table 6. Comparison of homogeneity of Chi square for transformed and non transformed data.

Experiment	Non Transformed	Transformed: Sums	Transformed: Individual Roots
I			
χ^2	7.60	0.54	
P_{χ^2}	0.06	0.89	
II			
χ^2	6.56	1.55	
P_{χ^2}	0.09	0.66	
III			
χ^2	7.06	5.15	2.06
P_{χ^2}	0.22	0.37	0.83
IV			
χ^2	3.33	4.77	1.45
P_{χ^2}	0.50	0.32	0.83

Table 7.

Source of Variation	df	F Values For: Non Transformed	F Values For: Transformed
Experiment V			
Total	11	--	--
Treatment	2	26.16**	48.79**
Error	9	8.20	0.53
Experiment VI			
Total	47	--	--
Treatment A	3	5.99**	5.06**
Treatment B	2	1.60	0.22
A X B	6	4.48**	0.34
Error	36	3.21	0.49

Table 8.

Comparison of t values for non transformed and transformed counts.

Experiment	t: Non Transformed	t: Transformed
A	1.63	2.20*
B	2.40*	2.45*
C	2.72**	3.48**
D	2.33*	2.94**
E	1.96	1.29
F	1.21	1.83
G	1.16	1.24
H	1.58	0.46

Table 9. Number of one root samples to discover a specific difference at different population means. Non transformed data.

Mean	0.5	1.0	2.0	3.0	5.0	8.0
St. dev.	1.7	2.2	3.0	3.8	5.2	7.3
Difference to be discovered	Number of Samples					
0.5 B/R	45	75	139	223	416	819
1.0 B/R	12	19	35	56	104	205
2.0 B/R		5	9	14	26	52

Table 10. Number of one root samples to discover a specific difference at different population means. Data transformed with inverse hyperbolic sine.

Mean	0.5	1.0	2.0	3.0	5.0	8.0
Trans. mean	0.25	0.45	0.71	0.91	1.23	1.59
Trans. st. dev.	0.52	0.62	0.70	0.73	0.71	0.62
Difference to be discovered	Number of Samples					
0.5 B/R	20	52	188	256	400	913
1.0 B/R	6	13	43	64	100	228
2.0 B/R		3	9	16	24	51

Table 11. Analysis of 1955 sampling study. Individual root counts transformed by inverse hyperbolic sine, q = 1.5.

Factor	Sum of Squares	Degrees Freedom	Mean Square	Component	Sample Units	Contribution to Variance
Total	446.0603	1279				
Range	13.4339	3	4.47797	0.01125	4	0.0028125
Block	10.5248	12	0.87707	0.00024	16	0.0000150
Plot	41.1733	48	0.85778	0.01854	64	0.0002897
Set	93.4884	192	0.48692	0.04124	256	0.0001611
Root	287.4399	1024	0.28070	0.28070	1280	0.0002193

Table 12. Analysis of 1956 sampling study. Individual root counts transformed by inverse hyperbolic sine, q = 3.0.

Factor	Sum of Squares	Degrees Freedom	Mean Square	Component	Sample Units	Contribution to Variance
Total	233.1874	959				
Field	10.6687	2	5.33435	0.01667	3	0.0055567
Range	0.7323	3	0.24410	0.00000*	6	0.0000000
Block	12.7657	6	2.12762	0.01978	12	0.0016483
Plot	19.6318	36	0.54533	0.00943	48	0.0001965
Set	51.3721	144	0.35675	0.03541	192	0.0001844
Root	138.0168	768	0.17971	0.17971	960	0.0001872

* Actual estimate = -0.01177

Table 13. Relative efficiency of various sampling designs for estimating sampling variance within a plot.

Alternative								
Year	Plots	Sets Per Plot	Roots Per Set	Total Roots	Variance	Relative Precision	Relative Cost	Relative Efficiency
1955	64	4	5	1280	.00067	1.00	1.00	1.00
	64	10	2	1280	.00057	1.17	1.25	0.94
	64	7	3	1344	.00059	1.13	1.17	0.97
	64	6	3	1152	.00065	1.03	1.01	1.02*
1956	64	4	5	1280	.00043	1.00	1.00	1.00
	64	10	2	1280	.00034	1.25	1.25	1.00
	64	7	3	1344	.00036	1.18	1.17	0.97
	64	6	3	1152	.00040	1.06	1.01	1.05*

* Computed from formula to minimize variance at a fixed cost.

Table 14. Relative efficiency of various sampling plans for estimating the population in an area composed of several fields.

Alternative									
Fields Per Area	Blocks Per Field	Plots Per Block	Sets Per Plot	Roots Per Set	Total Roots	Variance	Relative Precision	Relative Cost	Relative Efficiency
4	4	4	4	5	1280	.005828	1.00	1.00	1.00
4	8	2	20	1	1280	.005099	1.14	1.61	0.71
8	8	1	10	2	1280	.002732	2.15	1.38	1.56
14	6	1	2	3	504	.002106	2.76	0.89	3.10
16	4	1	10	2	1280	.001691	3.45	1.59	2.17
16	6	1	2	3	576	.001856	3.14	1.02	3.08*
32	4	1	10	1	1280	.000916	5.57	2.51	2.53

* Computed from formula to minimize variance at a fixed cost.

SAMPLING STUDIES

OF THE

CLOVER ROOT BORER APPENDIX TABLES

Table A. Distribution of borers in fields having a mean of 0 to 1 borer per root. Ohio survey, 1954.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial 1/	Calculated Frequency: Neyman $n \rightarrow \infty$
0	237	232.0	242.8
1	24	38.9	25.3
2	24	18.1	16.9
3	17	10.2	11.2
4	7	6.2)	7.4
5	2	4.0)	)
6	2		
7	3	8.6)	14.4)
8+	2		
	318	318.0	318.0
χ^2		12.71	8.23
P_{χ^2}		0.01	0.08

In this and subsequent tables, brackets indicate that these frequencies have been lumped for computing Chi-square.

Table B. Distribution of borers in fields having a mean of 1 to 1-1/2 borers per root. Ohio survey, 1954.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial	Calculated Frequency: Neyman $n \to \infty$
0	118	106.1	111.5
1	41	53.5	45.9
2	21	29.5	29.1
3	21	16.7	17.8
4	13	9.6	10.6
5	6	5.3	6.2
6	4	)	)
7	4	8.3)	7.9)
8	1	)	)
	229	229.0	229.0
χ^2		9.16	4.43
P_{χ^2}		0.16	0.48

Table C. Distribution of borers in fields having a mean of 1-1/2 to 2 borers per root. Ohio survey, 1954.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial	Calculated Frequency: Neyman $n \to \infty$
0	100	92.8	104.4
1	18	32.9	20.9
2	26	18.9	15.9
3	13	12.2	12.0
4	10	8.4	9.0
5	6	5.9	6.7
6	3	4.3)	5.0)
7	2	3.1)	3.7)
8	3	9.5)	10.4)
9+	7	)	)
	188	188.0	188.0
χ^2		11.14	8.85
P_{χ^2}		0.08	0.18

Table D. Distribution of borers in fields having a mean of 2 to 2-1/2 borers per root. Ohio survey, 1954.

Borers Per Root	Observed Frequency	Calculated Frequency Negative Binomial	Calculated Frequency Neyman $n \to \infty$
0	81	68.4	76.4
1	43	53.5	45.3
2	29	38.6	35.8
3	29	27.1	26.9
4	19	18.7	19.5
5	19	12.8	13.8
6	9	8.7	9.5
7	3	5.9	6.5
8	5	4.0	)
9+	9	8.3	12.3)
	246	246.0	246.0
χ^2		11.64	5.96
P_{χ^2}		0.17	0.54

Table E. Distribution of borers in fields having a mean of 2-1/2 to 3 borers per root. Ohio survey, 1954.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial	Calculated Frequency: Neyman $n \to \infty$
0	106	95.7	114.8
1	41	58.9	42.2
2	43	41.0	34.5
3	32	29.6	27.8
4	25	21.8	22.1
5	19	16.3	17.3
6	11	12.2	13.4
7	7	9.2	10.4
8	9	7.0	7.9
9	6	5.3	6.0
10+	15	17.0	17.6
	314	314.0	314.0
χ^2		9.32	6.04
P_{χ^2}		0.41	0.74

Table F. Distribution of borers in fields having a mean of 3 to 4 borers per root. Ohio survey, 1954.

Borers Per Root	Observed Frequency	Calculated Frequency	
		Negative Binomial	Neyman $n \to \infty$
0	97	75.9	92.2
1	53	72.9	58.8
2	52	61.8	52.8
3	46	45.0	45.5
4	47	35.6	37.9
5	38	27.7	30.8
6	24	21.4	24.6
7	16	16.4	19.2
8	11	12.5	14.9
9	14	9.5	11.3
10	5	7.2	)
11	8	5.4	)
12	4	4.1	41.0)
13+	14	33.6	)
	429	429.0	429.0
χ^2		36.34	9.35
P_{χ^2}		< 0.01	0.40

Table G. Distribution of borers in fields having a mean of 4 to 5 borers per root. Ohio survey, 1954.

Borers Per Root	Observed Frequency	Calculated Frequency	
		Negative Binomial	Neyman $n \rightarrow \infty$
0	134	130.2	121.6
1	45	75.9	59.9
2	47	55.1	53.8
3	47	42.6	47.2
4	46	33.9	40.5
5	36	27.5	34.3
6	36	22.5	28.6
7	24	18.6	23.5
8	21	15.5	19.2
9	14	12.9	15.5
10	8	10.8	12.5
11	6	9.1	
12	8	7.7	
13	7	6.5	45.4
14	6	5.5	
15+	21	31.7	
	506	506.0	506.0
χ^2		38.48	10.56
P_{χ^2}		< 0.01	0.39

Table H. Distribution of borers in fields having a mean of 5 to 6 borers per root. Ohio survey, 1954.

Borers Per Root	Observed Frequency	Calculated Frequency Negative Binomial	Neyman $n \rightarrow \infty$
0	57	44.6	67.6
1	23	37.3	26.6
2	30	31.9	16.1
3	27	27.1	21.6
4	24	23.1	19.3
5	23	19.6	17.2
6	20	16.7	15.3
7	16	14.2	13.5
8	13	12.0	11.8
9	12	10.2	10.3
10	5	8.7	8.9
11	9	7.4	)
12	4	6.2	)
13	4	5.3	71.8)
14	7	4.5	)
15+	26	31.2	)
	300	300.0	300.0
χ^2		16.23	29.23
P_{χ^2}		0.29	< 0.01

Table I. Distribution of borers in fields having a mean of 6 to 8 borers per root. Ohio survey, 1954.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial	Calculated Frequency: Neyman $n \to \infty$
0	36	20.5	33.2
1	17	26.8	23.3
2	26	28.4	24.1
3	22	27.7	23.9
4	26	25.9	23.0
5	22	23.5	21.6
6	22	21.0	19.9
7	16	18.5	18.1
8	17	16.1	16.2
9	15	14.0	14.4
10	13	12.0	12.6
11	10	10.3	11.0
12	12	8.7	9.5
13+	44	45.6	48.2
	299	299.0	299.0
χ^2		17.63	4.13
P_{χ^2}		0.12	0.98

Table J. Distribution of borers in fields having a mean of more than 8 borers per root. Ohio survey, 1954.

Borers Per Root	Observed Frequency	Calculated Frequency	
		Negative Binomial	Neyman $n \rightarrow \infty$
0	5	4.4	8.7
1	3	7.0	6.8
2	10	8.5	7.6
3	8	9.2	8.0
4	8	9.4	8.2
5	12	9.3	8.2
6	5	8.9	8.0
7	10	8.4	7.8
8	10	7.8	7.4
9	8	7.1	6.9
10	8	6.5	6.4
11	6	5.8	5.9
12	6	5.2	5.4
13	7	4.7	
14	6	4.1)	
15	3	3.7)	
16	1	3.2	
17	1	2.8)	38.7)
18	5	2.5)	
19	2	2.1)	
20	2	1.9)	
21+	8	11.5	
	134	134.0	134.0
χ^2		7.99	9.74
P_{χ^2}		0.89	0.64

Table K. Distribution of borers in 1955 sampling study. Wooster, Ohio.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial	Calculated Frequency: Neyman $n \to \infty$
0	423	395.9	443.5
1	227	275.7	225.8
2	206	190.3	174.8
3	133	131.0	130.5
4	83	90.1	94.7
5	70	61.8	67.2
6	49	42.4	46.9
7	33	29.1	32.2
8	20	20.0	21.8
9	9	13.7	14.7
10	6	9.4	9.7
11	6	6.4	
12	6	4.4)	18.2)
13	3	3.0)	)
14+	6	6.8	)
	1280	1280.0	1280.0
χ^2		18.29	12.46
P_{χ^2}		0.10	0.40

Table L. Distribution of borers in insecticidally treated plots. Wooster, Ohio, 1954-55.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial	Calculated Frequency: Neyman $n \rightarrow \infty$
0	1096	1036.7	1125.3
1	182	256.9	158.0
2	126	133.7	115.1
3	84	81.8	83.6
4	60	53.8	58.0
5	46	36.8	41.7
6	31	26.0	29.9
7	24	18.5	21.4
8	15	13.4	15.3
9	9	9.8	10.9
10	3	7.3	
11	5	5.4	
12	4	4.1	
13	1	3.1	37.7
14	4	2.4	
15	2	1.7	
16+	5	5.6	
	1697	1697.0	1697.0
χ^2		34.48	11.63
P_{χ^2}		< 0.01	0.31

Table M. Distribution of borers in Indiana tests, 1954.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial	Calculated Frequency: Neyman $n \to \infty$
0	43	45.5	41.4
1	9	7.2	10.1
2	4	3.1	4.7
3+	4	4.2	3.8
	60	60.0	60.0
χ^2		0.86	0.29
P_{χ^2}		0.83	0.96

Table N. Distribution of borers in Indiana tests, 1955.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial	Calculated Frequency: Neyman $n \to \infty$
0	359	344.2	360.0
1	49	67.0	45.7
2	27	31.2	29.7
3	21	17.4	19.2
4	10	10.5	12.4
5	9	6.6	7.9
6	7	4.3	5.1
7+	8	8.8	10.0
	490	490.0	490.0
χ^2		9.38	2.38
P_{χ^2}		0.22	0.91

Table 0. Distribution of borers in Indiana airplane test, 1955.

Borers Per Root	Observed Frequency	Calculated Frequency	
		Negative Binomial	Neyman $n \quad \infty$
0	49	50.6	58.4
1	16	15.8	8.7
2	14	9.1	6.3
3	6	6.1	5.5
4	3	4.3	4.3
5	2		
6	1	7.3	8.2
7	2		
8+	7	6.8	8.6
	100	100.0	100.0
χ^2		3.81	19.03
P_{χ^2}		0.69	< 0.01

Table P. Distribution of borers in check plots. Virginia, experiments 1 and 2, 1954.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial	Calculated Frequency: Neyman $n \rightarrow \infty$
0	8	7.2	8.6
1	10	12.3	11.8
2	7	14.2	13.4
3	23	13.8	13.2
4	14	12.2	11.9
5	10	10.2	10.1
6	12	8.0	8.2
7	4	6.2	6.4)
8	2	4.6)	)
9	1	3.4)	)
10	4	2.4)	16.4)
11+	5	5.5	)
	100	100.0	100.0
χ^2		14.51	14.86
P_{χ^2}		0.04	0.04

Table Q. Distribution of borers in insecticidally treated plots. Virginia, experiment 2, 1954.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial	Calculated Frequency: Neyman $n \to \infty$
0	227	219.7	222.0
1	35	50.2	45.7
2	23	17.8	19.2
3	12	7.1	7.9
4	1	5.2	5.2
5	1		
6	1		
	300	300.0	300.0
χ^2		10.67	6.43
P_{χ^2}		0.01	0.09

Table R. Distribution of borers in check plots. Virginia, experiments 1 - 4, 1955.

Borers Per Root	Observed Frequency	Calculated Frequency		
		Negative Binomial	Neyman $n = o$	Neyman $n \to \infty$
0	43	24.7	28.6	26.4
1	17	39.5	36.2	38.4
2	33	40.6	38.3	39.4
3	46	34.1	33.5	33.7
4	21	25.4	25.9	25.6
5	17	17.4	18.3	17.8
6	15	11.3	12.1	11.7
7	8	7.0	17.1	7.3
8+	10	10.0		9.7
	210	210.0	210.0	210.0
χ^2		34.07	24.59	29.78
P_{χ^2}		< 0.01	< 0.01	< 0.01

Table S. Distribution of borers in insecticidally treated plots. Virginia, experiment I, 1955.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial	Calculated Frequency: Neyman $n \to \infty$
0	207	195.1	203.9
1	38	49.2	37.0
2	14	23.3	23.0
3	19	12.7	14.2
4	5	7.4	8.7
5	6	4.5	5.3
6+	11	7.8	7.9
	300	300.0	300.0
χ^2		12.69	8.10
P_{χ^2}		0.03	0.14

Table T. Distribution of borers in insecticidally treated plots. Virginia, experiment 2, 1955.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial	Calculated Frequency: Neyman $n \rightarrow \infty$
0	319	312.3	318.5
1	35	48.2	38.2
2	24	19.3	20.4
3	11	9.2	10.8
4	5	4.8	5.7
5	4	2.7)	3.0
6+	2	3.5)	3.3
	400	400.0	400.0
χ^2		5.26	1.84
P_{χ^2}		0.40	0.87

Table U. Distribution of borers in insecticidally treated plots. Virginia, experiment 3, 1955.

Borers Per Root	Observed Frequency	Calculated Frequency		
		Negative Binomial	Neyman $n = o$	Neyman $n \to \infty$
0	226	195.0	221.5	205.7
1	27	68.8	30.8	54.1
2	27	35.2	33.7	34.4
3	30	19.9	25.9	21.6
4	20	11.8	16.5	13.4
5	9	7.2	9.6	8.2
6+	11	12.1	12.0	12.6
	350	350.0	350.0	350.0
χ^2		43.59	3.40	23.97
P_{χ^2}		< 0.01	0.63	< 0.01

Table V. Distribution of borers in insecticidally treated plots. Virginia, experiment 4, 1955.

Borers Per Root	Observed Frequency	Calculated Frequency		
		Negative Binomial	Neyman $n = o$	Neyman $n \to \infty$
0	219	199.5	221.4	208.5
1	19	47.1	16.4	34.7
2	21	22.1	20.0	21.7
3	20	12.0	16.9	13.5
4	7	7.1	11.2	8.4
5	4	7.1	6.3	5.2
6+	10	5.1	7.8	8.0
	300	300.0	300.0	300.0
χ^2		24.32	4.10	11.79
P_{χ^2}		< 0.01	0.53	0.04

Table W. Distribution of borers in insecticidally treated plots. Oregon.

Borers Per Root	Observed Frequency	Calculated Frequency: Negative Binomial	Calculated Frequency: Neyman $n \to \infty$
0	1325	1168.0	1346.7
1	115	231.2	90.5
2	79	126.5	74.5
3	67	84.1	64.7
4	45	60.7	54.7
5	51	45.9	46.2
6	31	35.9	39.0
7	48	28.5	33.0
8	26	23.1	27.8
9	22	18.9	23.5
10	16	15.6	19.8
11	15	12.9	16.7
12	15	11.0	14.1
13	10	9.3	11.8
14	12	7.7	10.0
15	5	6.6	8.4
16	8	5.6	)
17	3	4.8)	)
18	1	4.0)	)
19	6	3.5)	46.6)
20	7	3.1)	)
21	8	2.7)	)
22	3	2.3)	)
23+	10	16.1	)
	1928	1928.0	1928.0
χ^2		135.95	21.25
P_{χ^2}		< 0.01	0.16

Table X. Distribution of borers in red clover roots. Fulton, New York, 1956. Neyman's distribution for $n \to \infty$.

Borers Per Root	June Counts		July Counts*		August Counts	
	Obs.	Expect.	Obs.	Expect.	Obs.	Expect.
0	52	49.3	15	15.0	5	5.6
1	5	6.2	4	6.9	4	3.6
2	2	5.4	6	6.8	5	3.9
3	5	4.8	12	6.6	3	4.1
4	6	4.3	11	6.3	5	4.2
5	6	3.8	5	5.9	3	4.2
6	2	3.3	4	5.6	4	4.1
7	3	2.9	7	5.1	4	4.0
8	1	2.6	6	4.7	5	3.9
9	2	2.2	1	4.3	3	3.7
10	3	2.0	5	3.9	3	3.5
11	2		3		7	3.3
12	1		2		3	3.1
13	1	13.2	2	28.9	2	2.8
14	2		3		3	2.6
15+	7		14		20	22.4
	100	100.0	100	100.0	79	79.0
χ^2		3.87		12.33		3.23
P_{χ^2}		0.69		0.14		0.78

* 57.11425 used as estimate of variance for calculations instead of sample variance.